

Neural Network Graph Classifier

of Probability Distributions

by

Patrick Spohr

February 20th, 2024

Code

<https://github.com/p-spohr/NN-Graph-Classifier>

HTW Berlin

Master's Program

Mathematical Finance, Actuarial Science, and Risk Management

Department

Information, Communication, and Economics

Course

Deep Learning Seminar

Professor

Dr. Alla Petukhina

Table of Contents

1	Motivation and Objectives	2
1.1	Graph Classifier	2
1.2	Literature Review	3
1.3	Main Goals	3
1.4	Initial Problems	4
2	Data	5
2.1	CIFAR-10 Images	5
2.2	Scraped Graphs	6
2.3	Generated Graphs	7
3	Methods	10
3.1	Overview	10
3.2	Feed-Forward Neural Networks	10
3.3	Convolutional Neural Networks	11
3.3.1	CNNs vs FNNs	11
3.3.2	CNN and Max Pooling Layers	12
3.3.3	Rectified Linear Units	15
3.3.4	Cross-Entropy	15
3.3.5	Adam Optimizer	16
3.4	Python, Keras, and Libraries	17
3.5	Simplified Graph Classifier	18
3.6	Distribution Graph Classifier	20
4	Results	22
4.1	Simple Graph Classifiers	22
4.2	Distribution Graph Classifiers	25
4.3	Conclusions and Further Research	31
	References	33

1 Motivation and Objectives

1.1 Graph Classifier

In the age of information, those that can successfully and quickly interpret large amounts of data will have an advantage. The field where converting data into visual aids is called descriptive statistics and the most common method to do this is with graphs. This follows the adage, "A picture says a thousand words." With just a glance someone can understand and make connections about the data without needing to scan through a table of numbers. There are many different kinds of graphs: bar charts, pie charts, line graphs, scatter plots, and radar charts – to name a few. Within those charts they can be divided up into even more subtypes, like stacked bar charts or donut charts. From science to business, charts are the main way key parts of the data are shown to other researchers, customers, or investors.

The McDonald's 2022 Annual Report [8] has a total of 73 pages and 8 graphs. Each graph displays important information to the investor, like the figure which shows total revenues by segment on page 13. One can see that from 2020 to 2022 the revenue from international operated markets rose and fell, but overall, the revenue share from the different segments have remained static. Would it be beneficial to have access to these graphs immediately without needing to browse through the whole report? The ability to identify and then classify graphs instantly in the report without needing to read through it and snip out the graphs manually would save time and effort. The overall flow of this system would have three parts: Identify, Locate, and Organize. Identify would simply see if it is a graph or not and classify it into a type of graph. Once that is finished then the graphs can be located on a page and extracted. The last part would organize the charts for easy viewing. My focus is on the Identify part of this system and will be limited to building a model that answers two questions: Is this image a graph? What kind of graph is it? Regarding the second question, I will limit the scope even further because of the multitude of different graphs. The focus will be on classifying the probability distribution in a graph. Regarding the graphs, probability distribution is commonly shown with a line graph or histogram, so the graph type will be one or both of these types of graphs. This was chosen because of the novelty of the problem – no similar projects were found – and because of my current study of stochastics.

1.2 Literature Review

I will need to briefly mention the literature I searched for and the literature that guided this project. As mentioned earlier, this project is quite novel in its end goal since I did not find any research that specifically tried to classify graphs into probability distributions. There were projects that classified graphs into bar charts, pie charts, etc., but even then, those projects were not part of published research. When one tries to search for a neural network graph classifier or a similar pattern the first results are about Graph Neural Networks or GNNs, which are unrelated. As for image classification itself, there are a plethora of papers and resources. I personally learned a lot from a Stanford University course [7] and *An Introduction to Convolutional Neural Networks* from O'Shea and Nash [17].

1.3 Main Goals

As stated previously, my focus is on the identification and classification of graph images. This can be accomplished in two types of models where the first one classifies a graph image from a natural image (non-graph) and the second one classifies a graph image into its probability distribution. There are many probability distributions to choose from, but I selected the normal distribution, log-normal distribution, exponential distribution, and uniform distribution. I primarily choose these distributions because they are common and relatively distinct from each other. The reasoning behind this two-model process is to be able to produce a baseline result where it is known that graphs can be distinguishable from natural images. I predicted early on that graphs would be quite distinguishable from natural images and the accuracy of this model would be high, but as it will be discussed later, there is a second component to this problem specific to my data. These hurdles will be expanded upon in the next section.

In summary, the workflow of the project was split into four steps: acquire data, build and train simple graph classifiers, build and train distribution graph classifiers, and evaluate the models. Getting data had the potential to be the hardest step of this project because of the novelty of its goals to classify graph images into their probability distributions and I was partly correct. The next major obstacle was choosing the right model to build the machine learning models and the method in which each image would be classified. In the 'Methods'

section, I will go in depth to explain the process of how the models were built and the images classified.

1.4 Initial Problems

As image classification is a vibrant and bustling field, there is no shortage of natural images of various types of things – cars, airplanes, horses, cats, dogs, clothes, etc. One of the most common datasets for natural images is the CIFAR-10 [14] as it is well labeled and contains various images. The task of finding graph images was much more difficult. However, there was a suitable dataset of graphs on Kaggle from SunEdition [19] which met my requirements to build the first model, which simply classifies an image as a graph or not a graph. Unfortunately, this dataset does not contain classes with specific probability distributions and cannot be used for the second model – classifying the four chosen probability distributions in graphs.

The biggest issue was getting a dataset with the normal distribution (norm), log-normal distribution (lognorm), exponential distribution (exp), and uniform distribution (unif) as its four classes. As I saw it, this could be solved in two possible ways. First, I could have scraped the graphs from various online sources. However, this leads to a major issue of not only where to scrape the graphs but how to label them time efficiently. The dataset from SunEdition on Kaggle has some major drawbacks when wanting to classify the images into more specific classes since many graphs have more than one graph type in the image. These would be apparent in my own scraped graphs as well and that makes the correct classification impossible. An example of this would be a histogram of the normal distribution on one side with a pie chart on the other side. Additionally, the dataset includes ‘graph-like’ images which are depictions of graphs but have no numerical value and are closer to art or natural images. I could also mention that even in a graph type, like bar charts, there are subcategories – stacked bar charts, horizontal bar charts, vertical bar charts, etc. Were I try to scrape the graphs, then I would lose control of the variables and spend most of my time labeling each graph which would be impossible for the timeframe of the project. This leads to the second solution to my problem. I could generate the graphs.

Generating graphs with the aid of a graphing library was the most time effective solution

to acquire thousands of graphs with the four distributions in already-labeled classes. This is not the ideal solution since it biases the dataset towards the design conventions of the chosen graphing library and it lacks the variety seen online, but it can be highly randomized to mimic the potential graphs in various sources and the variables can be controlled to ensure all graphs have the potential to be classified. The most important consideration in this method is the time saved by generating the graphs. The daunting task of scraping and labeling 10,000 graphs can now be minimized into writing a script using a graphing library. The specifics of the three datasets used will be further explained in the subsequent section.

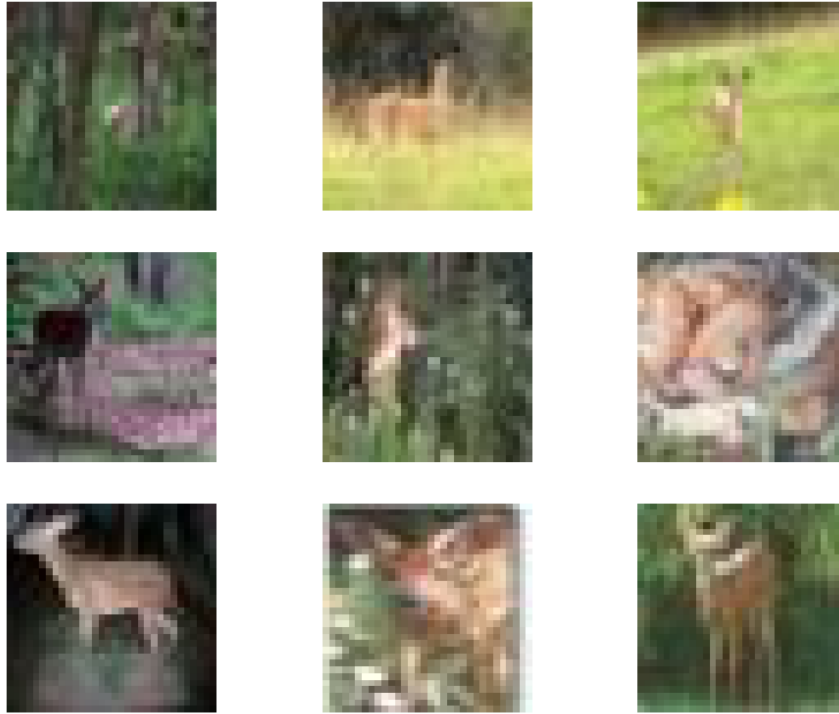
2 Data

2.1 CIFAR-10 Images

The CIFAR in CIFAR-10 stands for the Canadian Institute for Advanced Research and the 10 represents the 10 classes in the dataset: airplane, automobile, bird, cat, deer, dog, frog, horse, ship, and truck [14]. The data comes in six batches of 10,000 images with a total of 60,000 images. The 10,000 images in each batch are made up of 10 classes with 1,000 images per class. The images are colored and 32x32, which means they have a shape of (32,32,3). The first two values represent the dimensions and the 3 stands for the red, green, and blue channels. This data structure is known as a tensor. However, the original data type is a flat NumPy array of (3072,), so each image needed to be reshaped to fit in the model with the graphs.

The updated dataset has 10,000 images (first-batch) that are all 32x32 and in JPG format. Also, the 10 classes have been combined into one natural image class. I have only selected 10,000 because there are only 7,753 suitable graphs in the scraped graph dataset. Had I selected more than that would have led to an imbalanced dataset in the model. In my project, the CIFAR-10 dataset will be labeled as CIFAR or natural. See Visual 1 for examples.

Natural 32x32



Visual 1: CIFAR-10 Natural Images (CIFAR) Deer Examples [14]

2.2 Scraped Graphs

Just like the CIFAR-10 dataset, the scraped dataset [19] needed to get updated to work with the simple graph classifier model as well. The original dataset has 15,786 images and 8 classes. The classes are just image, bar chart, diagram, flow chart, graph, growth chart, pie chart, and table. The class ‘just image’ refers to non-graphs or natural images, so it needed to be removed. Other classes like table, flow chart, and growth chart were not necessary and were removed as well because tables have no visuals and the flow charts and growth charts are not based on numbers.

In the end, four classes were kept, bar chart, diagram, graph, and pie chart, so the total images now used were 7,753. The images came in various dimensions and were resized to the natural images 32x32 and converted to JPG format. I will note here that JPG was chosen

because it only has 3 color channels and no alpha channel. If PNG is used then it could have a shape of (32,32,4) which adds unnecessary parameters for the scope of my objectives. I'll refer to this dataset as scraped graphs or SCP. See Visual 2 for examples.

Scraped Graphs Original

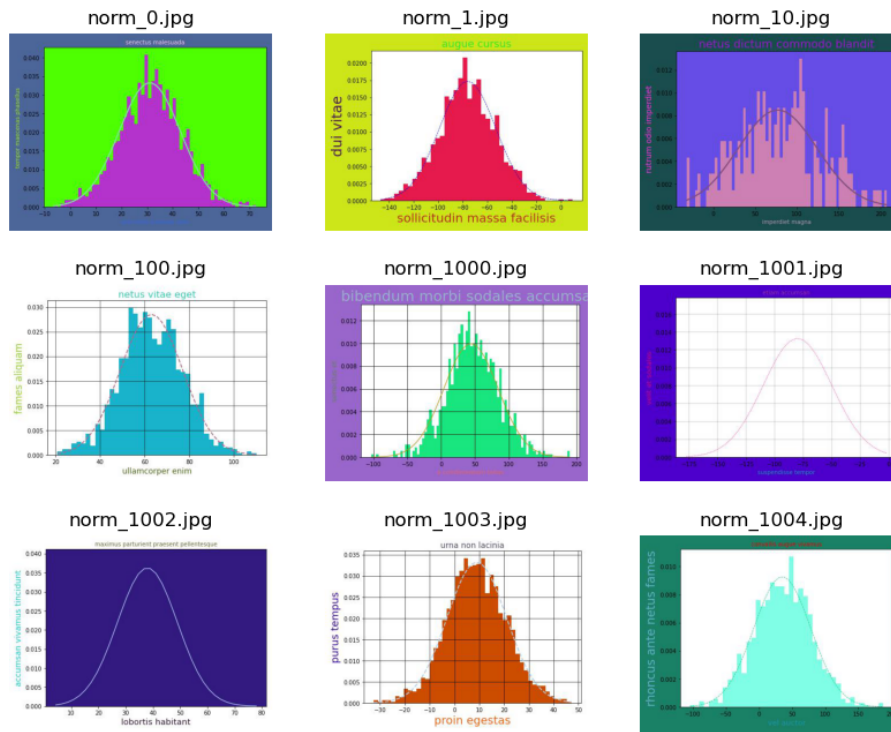


Visual 2: Scraped Graphs (SCP) Bar Chart Examples [19]

2.3 Generated Graphs

Lastly, 8000 graphs were generated in a 460x345 dimension and JPG format. The graphing library used was Matplotlib in Python, but I will discuss the programs and libraries in more detail in the “Methods” section. There were 4 classes – norm, lognorm, exp, and unif – with 2,000 images each. The only thing updated were the dimensions which included 32x32, 115x86, and 153x115. For clarity, these dimensions follow the Windows format of width by height.

Norm Distributions 460x345



Visual 3: Generated Graphs (GEN) Normal Distribution Examples

As for the design, almost every part of the graph was randomized. The color of the histogram bars, line, figure color (outside the box), face color (inside the box where the plot is), and text labels were changed, but the figure color and face color were biased to white by 20 percent because most graphs tend to be white. There was also some changes to ensure that the histogram bars, line, and face do not have the same color. Regarding the histogram and line, both were visible or only one was visible. The line style was solid or a dash. Each graph had a title, y label, and x label which consisted of lorem ipsum, which is Latin gibberish and is commonly used in web design. The idea behind this was to avoid a bias to English if someone used this model to predict graphs with different languages. The text also had randomized size. Lastly, the density function parameters were randomized to ensure many variations of each probability distribution.

Overall, the graphs looked like something that could be found in a report, book, or online. The biggest issue with the generated graphs were the so-called ‘duds’ which looked blank because the color of the face, histogram, and line were too similar. This was not a large amount of images, so I decided to keep them instead of manually deleting all of them. I reasoned that keeping the originals is better for reproducibility, rather than someone, me, deciding if a graph is a dud or not since some graphs are on the border of being a dud or not. Ultimately, the seed of the graph generator remains the same and the dataset can always be recreated and used in the model. See Visual 3 for examples.

Early on, I pre-identified possible misclassifications once I scanned through the set of generated graphs. The similarities between some of the graphs, especially the log-normal and normal distribution, were considerable. They were not ‘duds’ like previously mentioned – they looked typical, but because of the density functions’ parameters, they were similar in appearance to a point where it would become a 50-50 decision if I had to classify them myself. The cases when this happened are a minority, but it was important to keep this in mind as I evaluated the models later. The density function of each probability distribution used is listed below.

Normal Distribution Density Function $X \sim \mathcal{N}(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right]$$

Log-Normal Distribution Density Function $X \sim LN(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\log(x) - \mu)^2}{2\sigma^2}\right]$$

Exponential Distribution Density Function $X \sim \text{Exp}(\lambda)$

$$f_X(x) = \lambda \exp(-\lambda x)$$

Uniform Distribution Density Function $X \sim U[a, b]$

$$f_X(x) = \frac{1}{b - a}$$

3 Methods

3.1 Overview

As an introduction to the methods used, I will briefly list the steps taken to achieve my objectives. Firstly, I reviewed literature available on artificial neural networks (ANNs) and convolutional neural networks (CNNs). Then, I familiarized myself with TensorFlow, Keras, and Pillow (Python library for images). After that I found natural images (CIFAR) and scraped images (SCP). The next step was generating the graph dataset (GEN) in 32x32, 115x86, 153x115 dimensions. With all the data prepared, I trained simple graph classifiers using CIFAR, SCP, GEN datasets using 32x32 images. Those models were then evaluated by using saved Keras model to predict the images. Subsequently, I moved on to train the distribution graph classifiers using 32x32, 115x86, 153x115 dimensions. In order to see where the models of the distribution classifier models misclassified, I input all the images into the saved Keras models again. Lastly, I tested untrained images with the distribution classifier 153x115 which were not used in any model previously.

3.2 Feed-Forward Neural Networks

The ANN process used in my model is called a feed-forward neural network or FNN. The structure of the model consists of three parts and are shown in Visual 4: an input layer, one or many hidden layers consisting of neurons, and an output layer. The two main processes in the model are forward-propagation and back-propagation, and can be summed up by their respective goals, producing an estimate and minimizing the loss. Forward-propagation begins once the model is built and the data is split into the input values x and the target values y . In my simple model, a graph image x would be predicted to be y – a graph or a natural image.

The input data starts from the input layer and flows into the neurons in the hidden layers where it gets multiplied by a weight. The process can be defined as the dot product between the input values and their respective weights. The product then goes into an activation function in the neuron, so that the neuron ‘fires’, returning a modified value, or ‘rests’ –

gives zero. The process is repeated through every hidden layer until the output value from the final hidden layer enters the output layer, where it is again put into an activation to determine the final prediction or estimate. The estimate is then put into a loss function to calculate the loss or difference between the estimate and target value. From here a process called back-propagation starts which goes from the back to the front of the entire model. The loss function gets minimized by an optimizer using gradient descent to achieve the smallest possible loss and this is done by changing the weights between the neurons for every layer and every connection. Forward-propagation and back-propagation are repeated until the loss is minimized enough to suit the goal of the model.

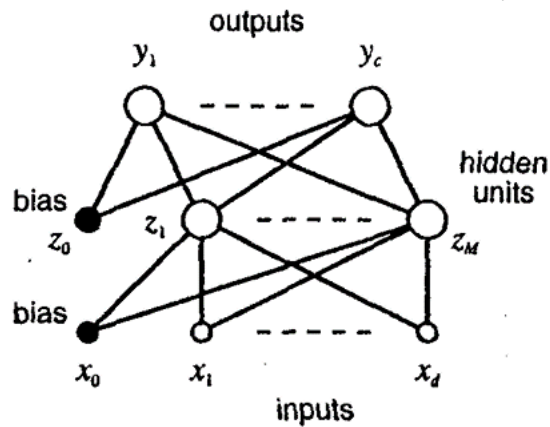


Figure 4.1. An example of a feed-forward network having two layers of adaptive weights. The bias parameters in the first layer are shown as weights from an extra input having a fixed value of $x_0 = 1$. Similarly, the bias parameters in the second layer are shown as weights from an extra hidden unit, with activation again fixed at $z_0 = 1$.

Visual 4: Neural Network [3]

3.3 Convolutional Neural Networks

3.3.1 CNNs vs FNNs

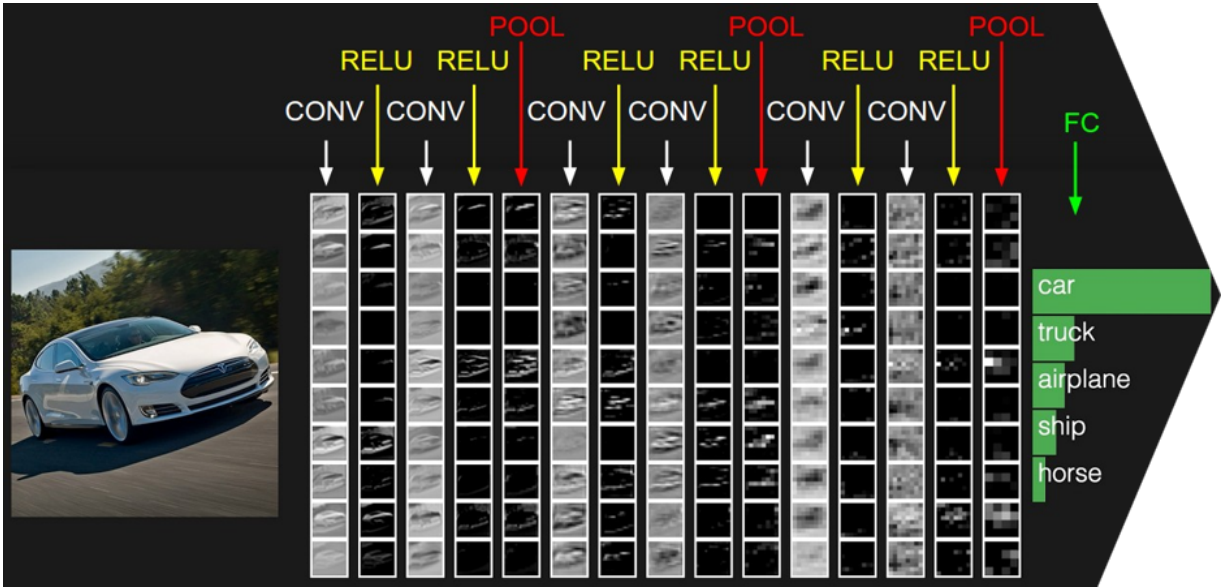
In order to classify images, the FNN model needs to be adjusted to take the tensor shape of images into account. This is where convolutional neural networks or CNN come into play.

The biggest difference between convolutional neural networks or CNNs and a more standard FNN is that they are mostly used in image classification. CNNs enable encoded image-specific features into the architecture (location + color), making the network more suited for image-focused tasks while also reducing the parameters required to set up the model [17]. Simply put, the overall structure of the model does not change, meaning the model still has an input layer, hidden layers, and an output layer. Additionally, the main processes of forward-propagation and back-propagation remain untouched, however the hidden layers get tweaked considerably whereby the neurons do not take one value, but a tensor and these neurons only connect to a small region of the layer preceding it. These new hidden layers are now called convolutional layers. The next major addition is another layer called a pooling layer, which comes after a convolutional layer.

3.3.2 CNN and Max Pooling Layers

The most important parts of a CNN are its convolutional layers and pooling layers since they are the means in which the original image gets ‘boiled down’ into its ‘essence’. I use this analogy because one can imagine a set of ingredients, pixels making up an image, that get thrown into a boiling pot where the water evaporates leaving only the important flavors behind. This visceral example can be summed up more literally, where the original image gets transformed layer by layer from the original pixel values to the final class scores. By the end, the full image will be reduced into a single vector of class scores, arranged along the depth dimension. What are these class scores? They indicate the estimate for each class or y target. As an example, the final output layer for CIFAR-10 would have dimensions of (1, 1, 10) since it has a total of 10 classes that an image can be sorted into [7] – airplane, dog, cat, etc. The step by step process can be seen more clearly in Visual 5, where the input image of car will be transformed into the most important features and then classified as a car.

What is a convolutional layer exactly? The four important parts are the filter, kernel, stride, and padding. These are the parameters that will determine how the convolution is performed. First, the number of filters are chosen, also known as output filters, and they determine how many times the image gets passed through the process. One filter means one pass through and one output tensor, while 10 filters mean ten pass throughs and 10 output



Visual 5: Long Short-Term Memory Unit [7]

tensors. The next choice is the size of the filter, which is known as the kernel. An example kernel size is (3,3,3) and this works just like image dimensions, where the height is 3, the width is 3, and the RGB channels are 3. The kernel in the example can be visualized as something that slides over the image and only focuses on a 3-by-3 area at a time. Adding on to the previous example, if the model has 10 filters, then that means there would be 10 output tensors, but that does not mean the output tensor shape would be (3, 3, 3). One needs the last two parameters, stride, and padding, to calculate the output tensor shape. The stride is a number that tells the kernel how many pixels it should move at a time over the image. A stride of one means the kernel moves across the image space 1 pixel at a time. The last parameter padding, or zero-padding, tells the model how many zeros to place around the image so that no vital information is lost when the kernel slides over the image. With the parameters decided, one can now calculate the output volume size or shape of the output tensor.

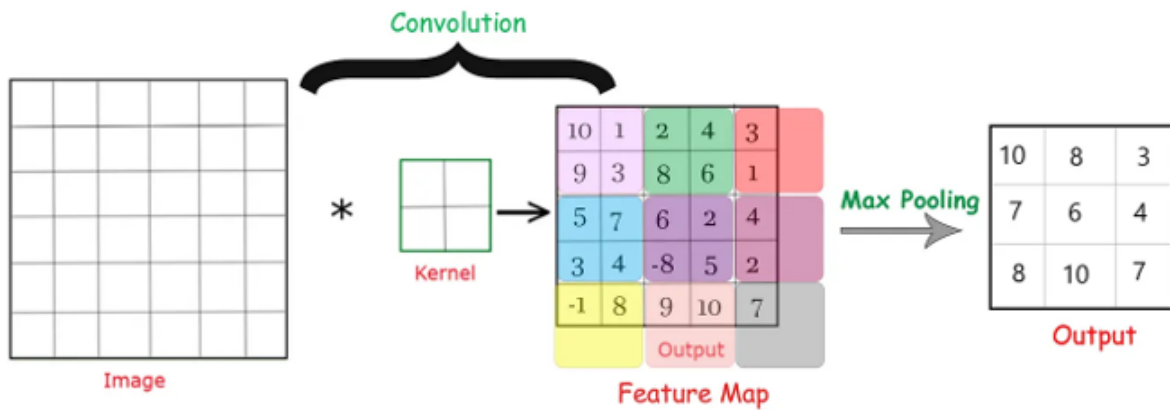
Output Volume Size Equation [7]:

$$V = \frac{W - F + 2P}{S} + 1$$

Where $W :=$ input volume size, $F :=$ receptive field size (kernel), $P :=$ amount of zero

padding, and $S :=$ stride of kernel

The convolutional layer is not the only new layer added in a CNN. The layer that often proceeds a convolutional layer is a pooling layer, or in the case of my model, a max pooling layer. The CNN and pooling layers work hand in hand to capture the most important parts of an image. Pooling is also known as downsampling and this is where a lower resolution version of an input signal is created. This method is used to retain the large or important structural elements without the fine detail that may not be as useful [4]. The outputs of the convolutional layers record the precise position of features but following small movements in the position of the feature, like a cat head slightly turned, will result in worse predictions, so max pooling is utilized, where only the highest value in every feature map or output filter goes on to the next layer. It can be noted that another pooling method, average pooling, can be used. It functions the same but averages the values in the feature patch.



Visual 6: Maxpooling Example [15] Credit: Codicals

Max Pooling Function:

$$M = \max(f_i)$$

for $i = 1, \dots, F$ feature map patches

3.3.3 Rectified Linear Units

As stated earlier, the activation function simply tells the neuron to ‘fire’ or ‘rest’ which in the case of the activation function I used, rectified linear unit function or ReLU, the value returned is either 0 if $x < 0$ or x if $x > 0$ [2]. ReLU is used as an activation function for a few key reasons. First, the simple nature of the function, $f(x) = \max(0,x)$, eliminates complex calculations and reduces the processing demands. This is especially important in convolutional layers where the amount of parameters can skyrocket and increase the time needed to learn. This works in practice by promoting sparsity in the model and sparsity refers to the scenario where most of the cell entries in a matrix, or in this case tensor, are zero [18].

Rectified Linear Units (ReLU):

$$f(x) = x^+ = \max(0, x) = \frac{x + |x|}{2} = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

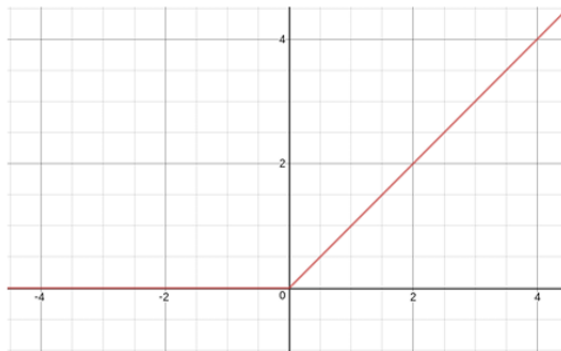


Figure 1: The Rectified Linear Unit (ReLU) activation function produces 0 as an output when $x < 0$, and then produces a linear with slope of 1 when $x > 0$.

Visual 7: Rectified Linear Unit [2]

3.3.4 Cross-Entropy

The next important topic to cover is the loss function used since the learning process can only happen if the model has a loss function to calculate the error. The loss function in the

models I built was the cross-entropy loss function. These functions, used in classification problems, calculate how accurate the machine learning or deep learning model is by defining the difference between the estimated probability with the desired outcome [21]. These functions come in two versions, one for binary classification and one for categorical classification. My model uses categorical cross-entropy because it is a multi-class classification problem, which means there is only one correct class per x input or image. Had I evaluated if multiple graphs were in an image, then it would become a multi-label classification problem, so the final prediction vector would have a dimensionality of the number of classes, but each element in the vector could be 0 or 1 depending if the image contains two classes [9]. For example, if an image had two graphs with one uniform distribution and one normal distribution, then it would look like [0, 0, 1, 1] (classes sorted alphabetically: exp, lognorm, norm, unif).

Binary Cross-Entropy Function:

$$BCEL = -\frac{1}{N} \sum_{i=1}^N (y_i \cdot \log(p_i) + (1 - y_i) \cdot \log(1 - p_i))$$

Categorical Cross-Entropy Function:

$$CCEL = -\frac{1}{N} \sum_{i=1}^N \sum_{j=1}^C (y_{i,j} \cdot \log(p_{i,j}))$$

Where N := number of rows

C := number of classes

y := indicator if class label j is the correct classification for observation i

p := predicted probability observation i is of class j

3.3.5 Adam Optimizer

To wrap up this section over CNNs, the optimizer used to minimize the loss function needs to be mentioned. The optimizer is called gradient descent and it can be described as a first-order iterative algorithm for finding a local minimum of a differentiable multi-variable function. This is apposed to gradient ascent where the algorithm tries to find the local maximum. This can be visually imagined as a rock falling from the mountain side where it will stop once it hits the lowest point or if it gets stuck in a low point between two high

points – which is to be avoided. The models use the Adam Optimizer and that is “...an algorithm for first-order gradient-based optimization of stochastic objective functions, based on adaptive estimates of lower-order moments [13].” The parameters used are alpha, beta1, beta2, and epsilon. Alpha is also known as the learning rate and determines the stepsize, and is usually very small at around 0.001, which is important, because the algorithm should not skip over the minimum. The major advantages of the Adam optimizer are that it only requires first-order gradients and has relatively little memory requirements.

Algorithm 1: *Adam*, our proposed algorithm for stochastic optimization. See section 2 for details, and for a slightly more efficient (but less clear) order of computation. g_t^2 indicates the elementwise square $g_t \odot g_t$. Good default settings for the tested machine learning problems are $\alpha = 0.001$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$. All operations on vectors are element-wise. With β_1^t and β_2^t we denote β_1 and β_2 to the power t .

Require: α : Stepsize

Require: $\beta_1, \beta_2 \in [0, 1)$: Exponential decay rates for the moment estimates

Require: $f(\theta)$: Stochastic objective function with parameters θ

Require: θ_0 : Initial parameter vector

$m_0 \leftarrow 0$ (Initialize 1st moment vector)

$v_0 \leftarrow 0$ (Initialize 2nd moment vector)

$t \leftarrow 0$ (Initialize timestep)

while θ_t not converged **do**

$t \leftarrow t + 1$

$g_t \leftarrow \nabla_{\theta} f_t(\theta_{t-1})$ (Get gradients w.r.t. stochastic objective at timestep t)

$m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$ (Update biased first moment estimate)

$v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$ (Update biased second raw moment estimate)

$\hat{m}_t \leftarrow m_t / (1 - \beta_1^t)$ (Compute bias-corrected first moment estimate)

$\hat{v}_t \leftarrow v_t / (1 - \beta_2^t)$ (Compute bias-corrected second raw moment estimate)

$\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \hat{m}_t / (\sqrt{\hat{v}_t} + \epsilon)$ (Update parameters)

end while

return θ_t (Resulting parameters)

Visual 8: Adam Optimizer Algorithm [13]

3.4 Python, Keras, and Libraries

The implementation of this project would not be possible without the various tools used. I will describe them all briefly and list out the versions used for the sake of reproducibility in Visual 9. TensorFlow [1] is an end-to-end platform for building machine learning models and that works with Keras [5], which is a deep learning API written in Python connected

to TensorFlow, to create all of the models in this project. Python comes with a plethora of libraries to make working with data and images easier. First there is NumPy [10] – a math library with powerful ndarrays. Then there is pandas [20] [16] which allows for the creation and manipulation of dataframes, which themselves are made up of NumPy’s arrays. The next important library is Matplotlib [11] where all of the generated graphs used in the models were plotted. Lastly, one of the most important libraires for image classification using TensorFlow and Keras is Pillow [6] since it allows the user to manipulate individual pixels of an image using Python. The PIL Image is a class that can be used and returned using TensorFlow.

Library	Versions
python	3.10.13
tensorflow	2.10.0
numpy	1.26.0
pandas	2.1.1
matplotlib	3.8.0
pillow	10.0.1

Visual 9: Python und Library Versions

3.5 Simplified Graph Classifier

In total, four simple graph classifiers were created – each with a specific purpose of evaluation in mind. The idea behind them was to create a base assumption that the graphs I generated can be considered a realistic dataset one could get if they scraped from various sources. The main goal is to be able to classify graphs into their probability distribution, but this step acts as a bridge and quality assurance for the generated graph dataset which will be used in the distribution graph classifiers. Each model has a specific name consisting of acronyms to distinguish itself: CIFAR is the natural images, SCP is the scraped graphs, GEN stands for the generated graphs, and 1 indicates the model structure. This results in the four models:

CIFAR_SCP_1, CIFAR_GEN_1, GEN_SCP_1, and CIFAR_GEN_SCP_1. The model structure for all classifiers is shown by layer in Visual 10.

Layer	Type	Parameters
1	Rescaling	(1/255, input_shape = (H, W, 3))
2	Conv2D	(filter_count = 16, kernal_size = 3, padding='same', activation = 'relu')
3	MaxPooling2D	none
4	Conv2D	(32, 3, 'same', 'relu')
5	MaxPooling2D	none
6	Conv2D	(64, 3, 'same', 'relu')
7	MaxPooling2D	none
8	Flatten	none
9	Dense	(units = 128, 'relu')
10	Dense	(number_of_classes = 4)

Visual 10: Model 1 Layer Information

Model Info	CIFAR_SCP_1	CIFAR_GEN_1
x	32x32 images	32x32 images
y	[graph, natural]	[graph, natural]
total	17,753	18,000
train	14,203	14,400
val	3,550	3600
model	1	1

Visual 11: Model Information for classifying graphs from natural images

The first model created aims to classify natural images from the scraped graphs. This answers the first basic question: how easily can the model distinguish a natural image from a graph image? The desired outcome would have a high accuracy since graphs have many combinations of characteristics that would not be found in natural images, for example grid

lines, large areas of only one color and surrounding text. One can imagine a natural image with some of these features, but this would be a small number of instances.

The second model, much like the first one, answers the question: are the generated graphs distinguishable from the natural images as well? This was the first test for the generated graphs and much like the first model, should have achieved high accuracy. The main question here would be the difference in accuracy between CIFAR_SCP_1 and CIFAR_GEN_1 since a higher accuracy in the former would indicate that my generated graphs have some similarities with natural images that are not apparent in the scraped graphs. This could be turned around and a much higher accuracy in CIFAR_GEN_1 could indicate that one of my original theories about the scraped dataset could be correct and that is the ‘graph-like’ nature of the images in the scraped dataset which are more artistic or non-numerical representations of graphs. In Visual 11, one can see that the input in CIFAR_SCP_1 and CIFAR_GEN_1 are balanced.

GEN_SCP_1 was a major test of the simple graph classifier since it will have indicated if my generated graphs were randomized enough to simulate what a bunch of scraped graphs would be. The desired outcome here would be a lower accuracy of around 50 percent to show the interchangeability of the two datasets. A high accuracy would indicate the need to randomize the generated graphs more and put them through more preprocessing steps.

Lastly, all of the datasets were used in a model to classify them as natural, generated, or scraped. The outcome of this would not be as important as the first three, but it would have reinforced the previous intermediate conclusions about the nature of the datasets. A very high accuracy would have meant all datasets are highly distinguishable from one another and point towards tweaking the generated graphs. The information for GEN_SCP_1 and CIFAR_GEN_SCP_1 can be seen in Visual 12.

3.6 Distribution Graph Classifier

The core of the project was being able to create a model that can classify graphs of four different probability distributions and they, like the simple graph classifiers, have specific naming conventions. DIST indicates that they are the distribution classifiers. Number x number specifies the dimensions of the image (width x height). The 1 indicates like earlier the first model structure. The three models are as follows: DIST_32x32_1, DIST_115x86_1,

Model Info	GEN_SCP_1	CIFAR_GEN_SCP_1
x	32x32 images	32x32 images
y	[generated, scraped]	[generated, natural, scraped]
total	15,753	25,753
train	12,603	20,603
val	3,150	5,150
model	1	1

Visual 12: Model Information for classifying generated from scraped graphs

and DIST_153x115_1. It is important to note that dimensions are reversed in TensorFlow and the shape of the image is (height, width, rgb).

Model Info	DIST_32x32_1	DIST_115x86_1	DIST_153x115_1
x	32x32 images	115x86 images	153x115 images
y	[exp, lognorm, norm, unif]	[exp, lognorm, norm, unif]	[exp, lognorm, norm, unif]
total	8,000	8,000	8,000
train	6,400	6,400	6,400
val	1,600	1,600	1,600
model	1	1	1

Visual 13: Model Information for all distribution graph classifiers

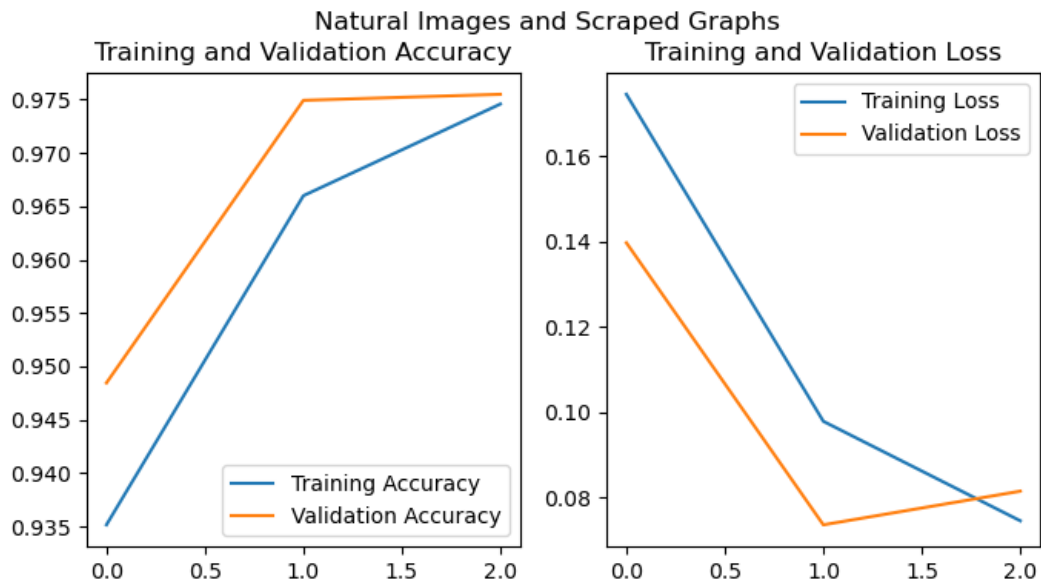
The images used, total number of images, training images, validation images, and model structure are all the same, except for the dimensions of the image and that can be seen in Visual 13. The reason for this is how much information is lost when the image is 32x32. It was easy to assume that the model would perform poorly when the density function line almost completely disappears. My solution was to increase the dimensions and thereby its quality. The classes, y target, are as previously stated: exp, lognorm, norm, and unif.

4 Results

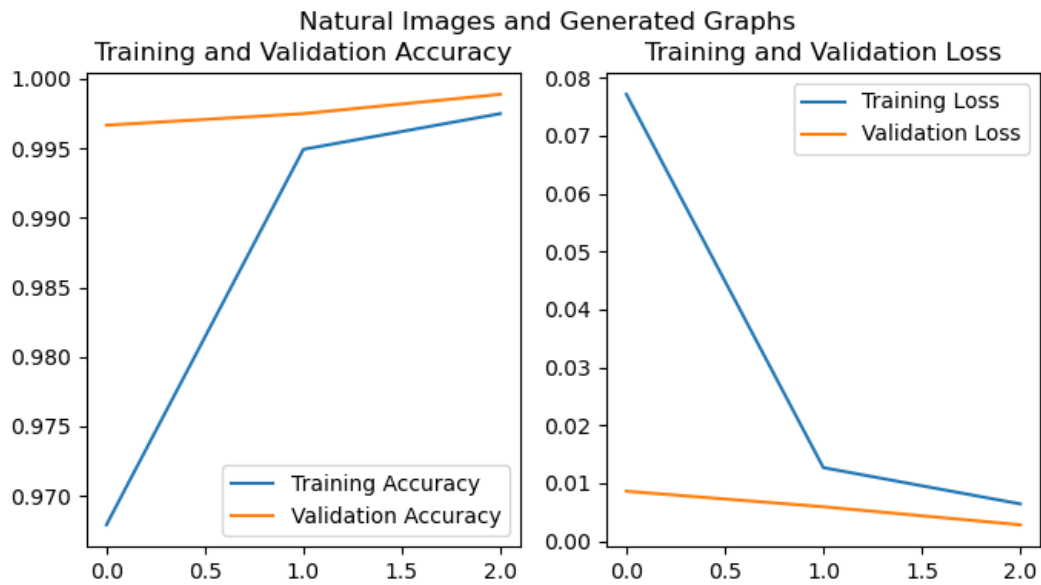
4.1 Simple Graph Classifiers

The first measure of all of the models were the training and validation accuracy and training and validation loss. The caveat here is the training validation should not be much higher than the validation accuracy because that indicates overfitting and future poor performance with untrained images. CIFAR_SCP_1 performed as expected and reached a 97.55 percent validation accuracy after 3 epochs. Subsequently, the CIFAR_GEN_1 performed slightly better with a final 99.89 percent validation accuracy after 3 epochs. Because of the high validation accuracy after 3 epochs, I felt it was unnecessary to increase this parameter more. The crucial model, GEN_SCP_1, achieved an accuracy of 99.21 percent, which as I mentioned, is the undesired outcome. This points to the previous hypothesis that the generated graphs and scraped graphs are so different that they can be considered disjunct subsets in the set of all graphs. This would mean the usability of models trained with the generated graphs are extremely limited in their capacity. The ramifications of this result indicate the need to tweak the programs that generate the graphs and the addition of preprocessing techniques. I will remark more on these additional techniques in the ‘Conclusions and Further Research’ section. The last simple graph classifier model, CIFAR_GEN_SCP_1 performed how the earlier models indicated with a 98.12 percent validation accuracy.

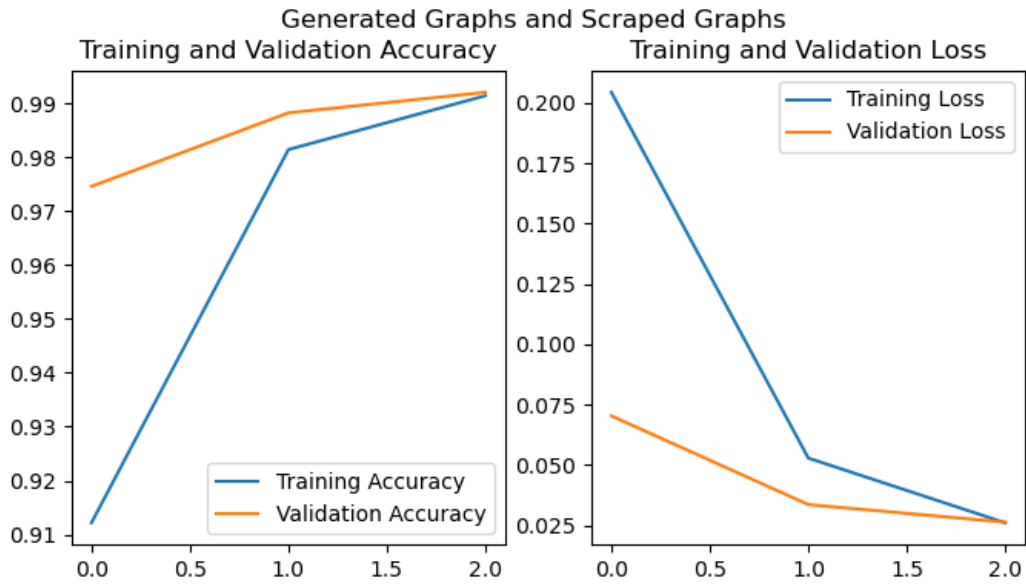
In order to properly evaluate the models, the accuracy and loss were not enough as indicators alone. To this end, I saved all of the models and fed in the graph images they have not seen yet. The models simply classified each image as natural or graph. First, I put the generated graphs in CIFAR_SCP_1. The perfect outcome should be 100 percent since all 8,000 images are graphs. The accuracy in this evaluation was 80.94 percent. This could be much better but considering the high accuracy from GEN_SCP_1, these graphs are not very similar. Next, the scraped graphs were put into CIFAR_GEN_1 and, like the above evaluation, should have a perfect accuracy of 100 percent. The accuracy dropped even lower to 37.22 percent which is far from ideal. What does this mean exactly? When graphs from the scraped dataset are put into the CIFAR_GEN_1, it classifies most of them as natural images. This outcome strengthens a hypothesis that quite a lot of scraped graphs are



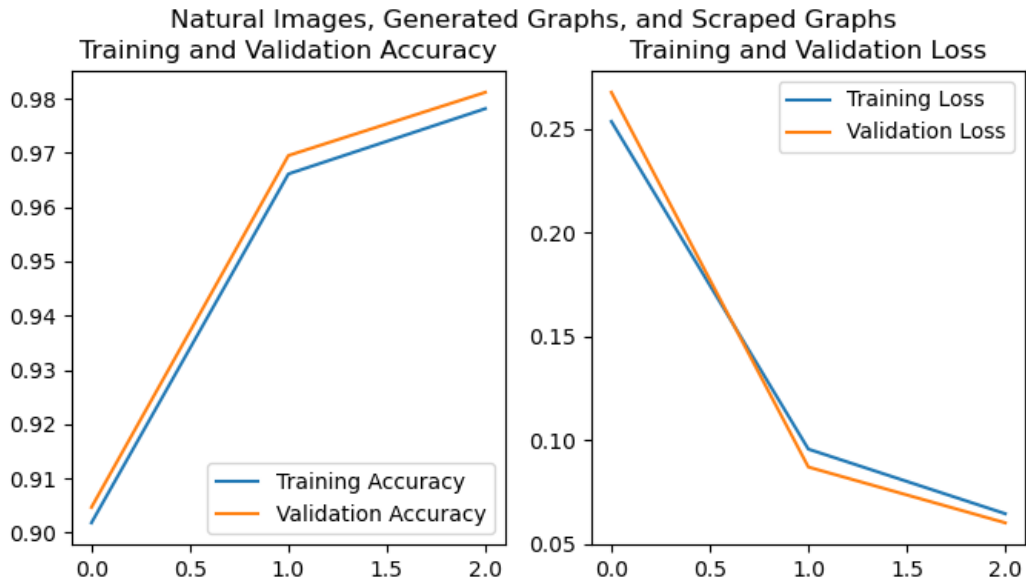
Visual 14: CIFAR_SCP_1 Training and Validation Accuracy and Loss



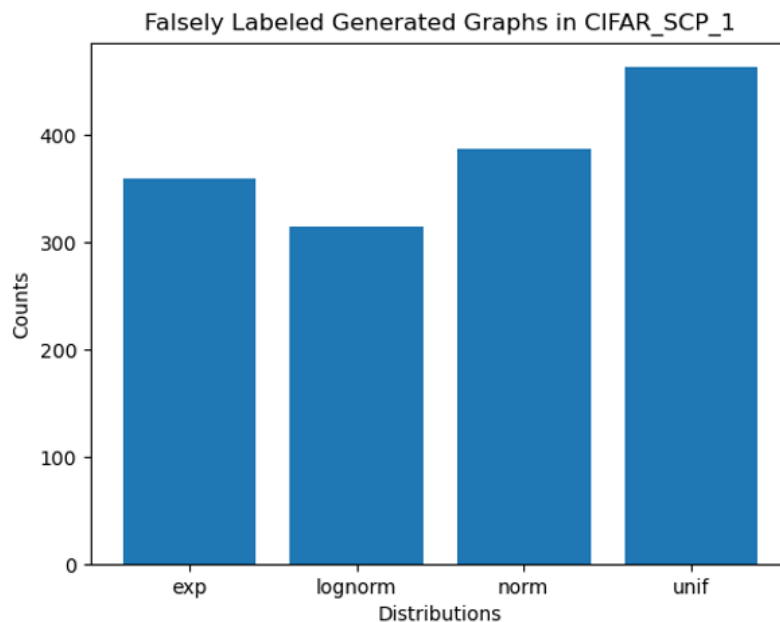
Visual 15: CIFAR_GEN_1 Training and Validation Accuracy and Loss



Visual 16: GEN_SCP_1 Training and Validation Accuracy and Loss



Visual 17: GEN_SCP_1 Training and Validation Accuracy and Loss

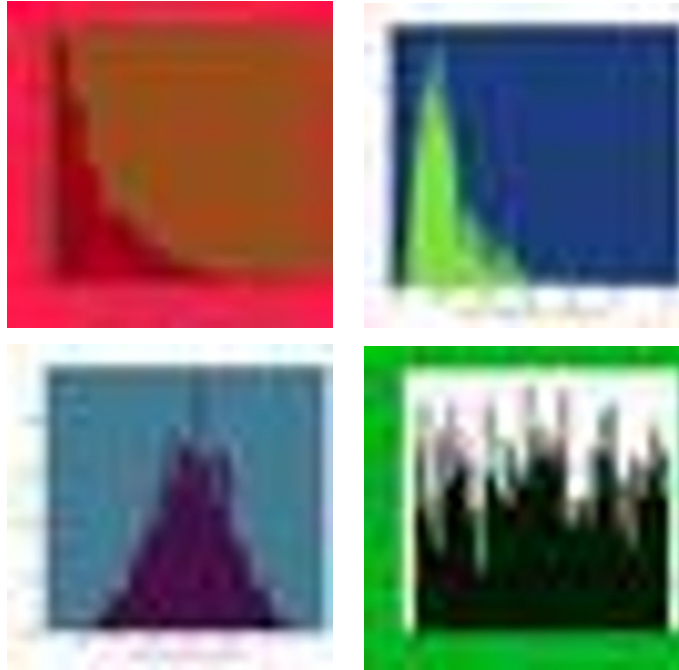


Visual 18: Misclassified generated graphs in CIFAIR_SCP_1

‘graph-like’ and are more similar to natural images. As the generated graphs are the main focus, I took a deeper look at the falsely classified generated graphs in CIFAR_SCP_1. The misclassifications were spread out pretty evenly, but the graphs with uniform distribution were most often classified as natural images. The counts can be seen in Visual 18 and some examples of misclassified graphs are shown in Visual 19.

4.2 Distribution Graph Classifiers

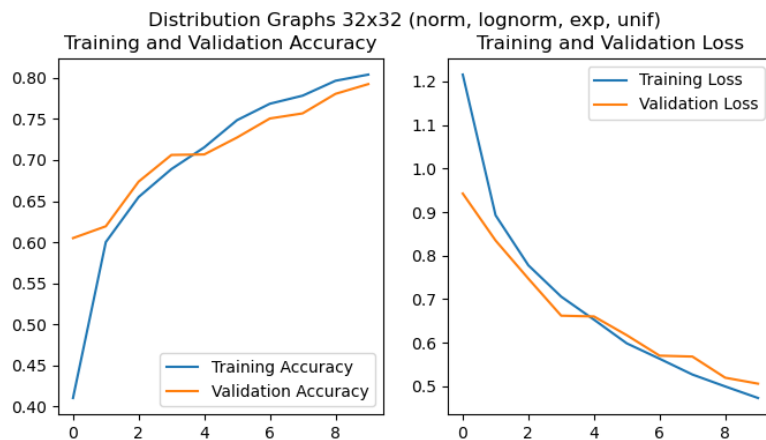
Just as before, I will first cover the training and validation accuracy and training and validation loss of each distribution graph classifier first. I will start with image data from the lowest dimension to the highest. The DIST_32x32_1 ran for 10 epochs since the model originally struggled to classify the graphs in only 3 epochs like the simple graph classifiers. This is a reasonable change in parameters since it is no longer a binary classification problem but a multi-class classification problem. After training it reached an accuracy of 79.25 percent validation accuracy. At an image size of 32x32 pixels this accuracy surprised me, especially when you can see that after one epoch the validation accuracy was already above 60 per-



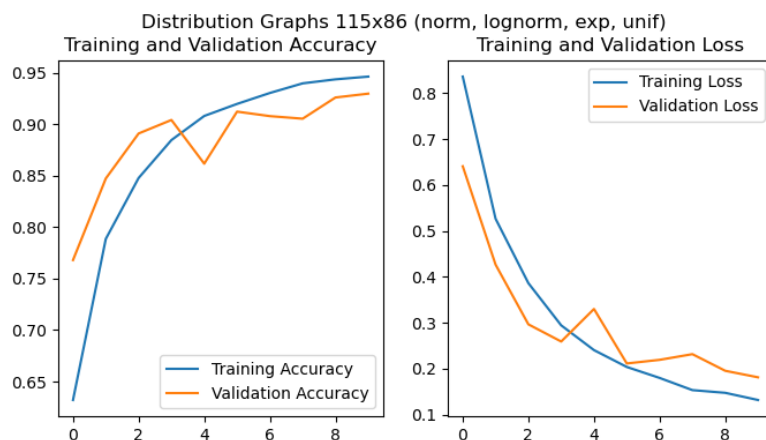
Visual 19: Examples of misclassified generated graphs as natural images

cent. Increasing the quality, or dimensions, of the image greatly increased accuracy. The DIST_115x86_1 was able to achieve a validation accuracy of 93.00 percent after 10 epochs. However, there is a diminishing return in regard to image quality since the validation accuracy only improved by 0.5 percent to 93.5 percent in the model DIST_153x115_1. Overall, the results are promising, but they require an evaluation to see where the graphs are getting misclassified. The validation accuracy of DIST_115x86_1 and DIST_153x115_1 are hanging around 93 percent, so the question becomes what are the images that are not getting classified correctly.

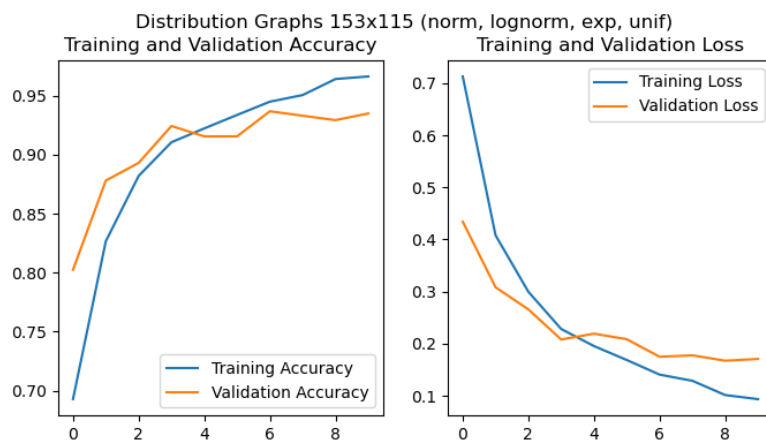
After the training was finished, I saved all the models and fed all 8,000 graphs into the DIST_153x115_1 for evaluation. I decided to stick with evaluating one model since doing the same process for all would have been redundant and not given me further insights into the data and model performance. I chose DIST_153x115_1 instead of the other two because it performed slightly better but using DIST_115x86 would have been viable as well. The saved DIST_153x115_1 model performed well in the evaluation by accomplishing an accuracy of 96.54 percent. That means there were a total of 270 misclassifications out of 8,000 images, and the results can be seen in Visual 23 and Visual 24. The probability distribution with



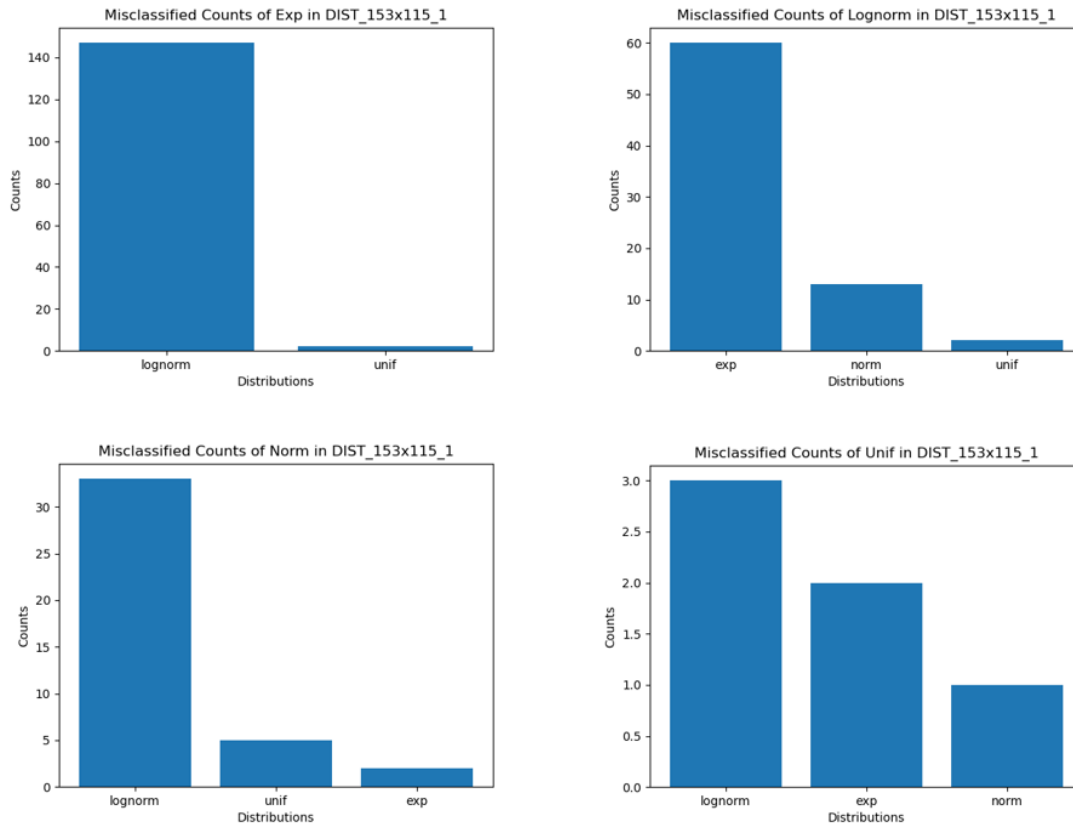
Visual 20: DIST_32x32_1 Training and Validation Accuracy and Loss



Visual 21: DIST_115x86_1 Training and Validation Accuracy and Loss



Visual 22: DIST_153x115_1 Training and Validation Accuracy and Loss



Visual 23: Counts of misclassified generated graphs in DIST_153x115_1

the most misclassifications was the exponential distribution at 149, while the lowest was the uniform distribution with only 6 misclassifications. Why were graphs with an exponential distribution most often labelled incorrectly and by such a wide margin? A closer look at the data in Visual 23 shows that almost all of the exponential distribution graphs that were misclassified were misclassified as the log-normal distribution. When one looks at the images in question and sees the similarities between the log-normal and exponential distribution graphs, then it is easier to see why that is the case, but it is still noteworthy how high the accuracy is despite the similarities. There were 2,000 exponential distribution graphs and only 147 were labeled as log-normal distribution which still indicates a strong performance.

In order to understand the results better and have a clear record of all the evaluations, I created CSVs with relevant information. They include the ending prediction weight for

Prediction	Exp	Lognorm	Norm	Unif
True	1,851	1,925	1,960	1,994
False	149	75	40	6

Visual 24: Counts of correctly and falsely classified generated graphs in DIST_153x115_1

each class, the true or false prediction, the file name, the max weight, and the label based on the max. With the results readily available, it was easy to see some patterns. In my earlier caveats, it was stated that the normal and log-normal distribution can look quite similar if given the right parameters and that plays out when one looks at how the model misclassified some normal distribution graphs as log-normal. Also, from the earlier section about the generated graphs, I mentioned the so-called ‘duds’ and that comes up in the evaluations as well. All of the uniform distribution graphs misclassified can be considered ‘duds’ in that they appear blank.

To wrap up my evaluations and to truly assess my generated graphs and models, I needed to use graphs that have not been used in any model previously. Visual 25 contains all images in the final test evaluation that were misclassified. To get this, I simply gathered 20 graphs of every distribution type from random sources online and then added one graph of each distribution drawn by hand. I felt this would add some realistic grounding to the final evaluation. Each distribution had 21 graphs, so a total of 84 graphs. The final evaluation accuracy was 59.10 percent and fell in line with my more hopeful expectations. This evaluation is similar to when the scraped graphs were put into CIFAR_GEN_1 and if you can remember, the accuracy was only 37.22 percent. So overall, it is a positive development. Just like before the question becomes what was getting misclassified and here is where the results diverged from my original predictions. Almost all of the misclassified graphs were misclassified as log-normal, which can be seen in the ‘label’ column in Visual 25. And if one analyzed this more granularly then one can see that the uniform distribution graphs were overwhelmingly misclassified as log-normal. 13 graphs out of the 21 uniform graphs were misclassified as log-normal. It is surprising how 61.90 percent of uniform graphs in the test dataset were misclassified as log-normal since the generated uniform graphs were predicted

exp	lognorm	norm	unif	max	file_name	actual	label	prediction
-4.1661463	8.161812	0.9788155	-10.229221	8.161812	exp_12.jpeg	exp	lognorm	False
3.8511171	4.496506	-3.405139	-5.2995405	4.496506	exp_2.jpeg	exp	lognorm	False
5.872565	11.971514	-6.275291	-12.128517	11.971514	exp_20.jpeg	exp	lognorm	False
9.207311	10.764248	-11.954131	-5.3854074	10.764248	exp_3.jpeg	exp	lognorm	False
-5.917483	15.367903	-4.7242074	-5.200668	15.367903	exp_5.jpeg	exp	lognorm	False
6.1337843	6.400875	-5.021023	-7.4242744	6.400875	exp_8.jpeg	exp	lognorm	False
0.51164746	8.585008	-3.2878444	-6.293257	8.585008	exp_9.jpeg	exp	lognorm	False
-2.7664876	8.546496	0.33398134	-9.835137	8.546496	exp_hand_0.jpeg	exp	lognorm	False
-13.257631	5.875923	-2.4565024	7.38458	7.38458	lognorm_0.jpeg	lognorm	unif	False
3.7302907	2.0135944	-0.048096493	-11.51519	3.7302907	lognorm_5.jpeg	lognorm	exp	False
-14.977435	3.678837	1.5030111	1.7347182	3.678837	norm_1.jpeg	norm	lognorm	False
-0.31633773	5.5405626	1.6448251	-18.00231	5.5405626	norm_11.jpeg	norm	lognorm	False
-14.40909	10.438942	7.7604384	-26.327517	10.438942	norm_13.jpeg	norm	lognorm	False
-5.3496103	7.2749767	1.1707792	-6.384265	7.2749767	norm_14.jpeg	norm	lognorm	False
-6.992462	7.189061	4.9432883	-15.288852	7.189061	norm_15.jpeg	norm	lognorm	False
-4.783489	4.332031	3.8387465	-14.778937	4.332031	norm_19.jpeg	norm	lognorm	False
-23.318886	12.883771	-0.8641091	5.143599	12.883771	norm_4.jpeg	norm	lognorm	False
-25.816198	16.998684	4.5914965	-9.728744	16.998684	norm_8.jpeg	norm	lognorm	False
-6.6115365	7.848427	0.57640886	-4.26034	7.848427	norm_9.jpeg	norm	lognorm	False
-0.7095418	1.1841375	-0.53598833	-1.6236649	1.1841375	unif_0.jpeg	unif	lognorm	False
-4.572587	5.3358355	0.4110088	-3.925213	5.3358355	unif_1.jpeg	unif	lognorm	False
-2.4820209	-0.05105117	2.7819424	-4.3270063	2.7819424	unif_10.jpeg	unif	norm	False
-4.820385	9.742123	-7.255406	6.477165	9.742123	unif_11.jpeg	unif	lognorm	False
-0.5606266	6.14713	-2.2170813	-3.020845	6.14713	unif_12.jpeg	unif	lognorm	False
1.1152176	1.1827534	-0.67093164	-5.319949	1.1827534	unif_13.jpeg	unif	lognorm	False
-0.19624099	7.653762	-2.4959579	-3.569571	7.653762	unif_14.jpeg	unif	lognorm	False
-0.46984562	5.252478	-4.9778605	1.5037757	5.252478	unif_16.jpeg	unif	lognorm	False
-0.20320779	4.1991606	1.5740888	-10.120164	4.1991606	unif_17.jpeg	unif	lognorm	False
-2.3884668	3.9697368	-0.6534233	-2.8202808	3.9697368	unif_2.jpeg	unif	lognorm	False
1.6306777	6.3168035	-0.7305184	-8.0671625	6.3168035	unif_3.jpeg	unif	lognorm	False
-5.6656113	4.946749	1.8187	-6.4847775	4.946749	unif_4.jpeg	unif	lognorm	False
-4.4202156	3.0252044	1.0488026	-2.0555344	3.0252044	unif_5.jpeg	unif	lognorm	False
5.331371	4.629284	-3.9633603	-6.628367	5.331371	unif_6.jpeg	unif	exp	False
-4.064364	4.5016537	-3.487114	3.3282218	4.5016537	unif_8.jpeg	unif	lognorm	False
-6.592189	-0.6548197	2.7110033	-1.7038764	2.7110033	unif_9.jpeg	unif	norm	False
0.5345101	0.08001919	0.43601054	-6.1528854	0.5345101	unif_hand_0.jpeg	unif	exp	False

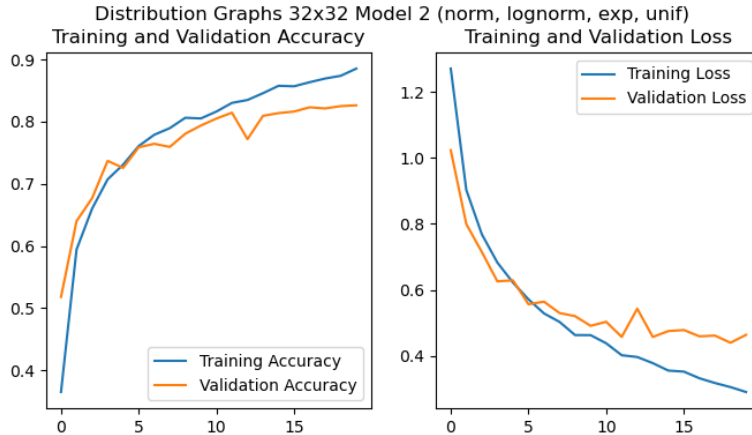
Visual 25: All Misclassified Images in Final Test Evaluation

so accurately in DIST_153x115_1. I can conclude for now that the model needs more diverse training images to account for the variety in graphs found online and more images to train on.

4.3 Conclusions and Further Research

As a whole, I can say that the main goals were accomplished. I have a model that can successfully identify graphs from natural images with greater than 98 percent accuracy and another model that can successfully identify my generated graphs with four different distribution types with around 93 percent accuracy. However, when more diverse graphs are included this drops down to 59.10 percent as shown in the final test evaluation and in Visual 25. Addendum to these results is the caveat from earlier that the generated graphs could be considered as a disjunct subset, if one considers the scraped graphs to be a genuine sample of possible and currently available graphs. The GEN_SCP_1's high accuracy points towards this theory but a large limiting factor is the small sample size – only 8,000 generated graphs and 7,753 scraped graphs. Some results that surprised me were that scraped graphs in CIFAR_GEN_1 were misclassified as natural images around 63 percent of the time. This could indicate that the generated graphs do not consider the more chaotic nature of graph design – this is substantiated by the final test evaluation as well. It would be safe to say that the generated graphs are very conservative in appearance. However, this was an original design choice to reduce the amount of variables in the graphs design, like not having two graphs in one image or keeping to the standard of having a complete graph in each image. Many graphs, like the ones in the scraped dataset and the ones I used in the final test evaluation, do not have any axes, labels, or titles. This enforces the idea that many of the images in SCP and many graphs in general are more ‘graph-like’ and not graphs in a very defined sense. Another surprise was the difference in how the uniform distribution graphs were getting misclassified. Uniform distribution graphs in DIST_153x115_1 were classified accurately around 99 percent of the time, but 20 percent of generated uniform graphs in CIFAR_SCP_1 were misclassified as natural images. Why were the graphs easily classified in DIST_153x115_1 but not in CIFAR_SCP_1? Perhaps graphs with the uniform distribution are unique from the other distribution graphs but not very unique from natural images. The final test eval-

uation bucked this trend by classifying uniform distribution as log-normal for a majority of the graphs. However, in the final test evaluation almost all of the misclassified images were classified as log-normal, so this might point to an issue with the training images.



Visual 26: DIST_32x32_2 Training and Validation Accuracy and Loss

To conclude I will propose some possible changes and improvements to further this line of research. The quickest improvements could simply come from changing the model structure and tweaking the parameters. While I only used one convolutional layer at a time, another common CNN architecture is to stack two convolutional layers before each pooling later. This is strongly encouraged as this allows for more complex features of the input vector to be selected [17]. Regarding the parameters, I built a second model by doubling the number of filters in the convolution layers to see what some intial changes to the model could do. The training and validation accuracy and loss of DIST_32x32_2 are shown in Visual 26 – the 2 indicates the second model used. There were minor improvements of the validation accuracy compared to the DIST_32x32_1, but the issue of overfitting is seen after 10 epochs. Secondly, one could use more images to train each model. It would be quite easy to generate 20,000 graphs, but that depends on the quality and ‘generalness’ of the graphs. The generated graphs used in this experiment were highly distinguishable from the scraped graphs and that leads to the next change, and that is improving the generated graphs. The scripts that created the graphs can be improved to increase variability, but another way is adding more preprocessing layers. TensorFlow has a host of different methods that change brightness,

contrast, crop, flip, and rotation, which would do a good job at simulating the chaotic nature of graphs created by people from all over the world. Finally, the experiment could be further by simply expanding the probability distributions used.

As stated at the beginning, the focus was on identifying and classifying graphs, but it was just the first step in a large picture of identifying, locating, and organizing. To extend a practical use case for this project, one could start using entire reports as input to train the model. In this scenario, pages with graphs on them would be the y target. With the pages identified, they could be returned to the user for quick viewing. From there the model could be expanded to only retrieve the graphs themselves and organize them into types. This would require more advanced techniques, especially those in the field of explainable AI [12], to target specific parts on a page and be able to extract more information.

References

- [1] Martin Abadi et al. “TensorFlow: A system for large-scale machine learning”. In: *12th USENIX Symposium on Operating Systems Design and Implementation (OSDI 16)*. 2016, pp. 265–283. URL: <https://www.usenix.org/system/files/conference/osdi16/osdi16-abadi.pdf>.
- [2] A. F. Agarap. “Deep Learning using Rectified Linear Units (ReLU)”. In: (3rd International Conference for Learning Representations). San Diego, 2019. URL: <https://arxiv.org/abs/1803.08375>.
- [3] Christopher M. Bishop. “Neural Networks for Pattern Recognition”. In: (1995). URL: <https://people.sabanciuniv.edu/berrin/cs512/lectures/Book-Bishop-Neural%20Networks%20for%20Pattern%20Recognition.pdf>.
- [4] Jason Brownlee. *A Gentle Introduction to Pooling Layers for Convolutional Neural Networks*. July 2019. URL: <https://machinelearningmastery.com/gentle-introduction-backpropagation-time/>.
- [5] François Chollet et al. *Keras*. 2015. URL: <https://keras.io>.

- [6] Alex Clark. *Pillow (PIL Fork) Documentation*. 2015. URL: <https://buildmedia.readthedocs.org/media/pdf/pillow/latest/pillow.pdf>.
- [7] *Convolutional Neural Networks for Visual Recognition*. July 2019. URL: <https://cs231n.github.io/convolutional-networks/>.
- [8] McDonald’s Corporation. *2022 Annual Report*. 2022. URL: https://corporate.mcdonalds.com/content/dam/sites/corp/nfl/pdf/MCD_2023_Annual_Report.pdf.
- [9] Raúl Gómez. *Understanding Categorical Cross-Entropy Loss, Binary Cross-Entropy Loss, Softmax Loss, Logistic Loss, Focal Loss and all those confusing names*. Mar. 2018. URL: https://gombru.github.io/2018/05/23/cross_entropy_loss/.
- [10] Charles R. Harris et al. “Array programming with NumPy”. In: *Nature* 585.7825 (Sept. 2020), pp. 357–362. DOI: 10.1038/s41586-020-2649-2. URL: <https://doi.org/10.1038/s41586-020-2649-2>.
- [11] John D. Hunter. “Matplotlib: A 2D graphics environment”. In: *Computing in Science & Engineering* 9.3 (2007), pp. 90–95. DOI: 10.1109/MCSE.2007.55.
- [12] V. Kamakshi and N. C. Krishnan. “Explainable Image Classification: The Journey So Far and the Road Ahead”. In: (Aug. 2023). DOI: <https://doi.org/10.3390/ai4030033>. URL: <https://arxiv.org/pdf/1811.03378>.
- [13] D. P. Kingma and J. L. Ba. “Adam: A Method for Stochastic Optimization”. In: *ICLR* (2015). URL: <https://arxiv.org/abs/1412.6980>.
- [14] A. Krizhevsky, V. Nair, and G. Hinton. *The CIFAR-10 Dataset*. Canadian Institute For Advanced Research. 2009. URL: <https://www.cs.toronto.edu/~kriz/cifar.html>.
- [15] P. Kumar and Codicals. *Max Pooling, Why use it and its advantages*. Aug. 2021. URL: <https://medium.com/geekculture/max-pooling-why-use-it-and-its-advantages-5807a0190459>.
- [16] Wes McKinney. “Data Structures for Statistical Computing in Python”. In: *Proceedings of the 9th Python in Science Conference*. Ed. by Stéfan van der Walt and Jarrod Millman. 2010, pp. 56–61. DOI: 10.25080/Majora-92bf1922-00a.

- [17] Keiron O'Shea and Ryan Nash. *An Introduction to Convolutional Neural Networks*. 2015. URL: <https://arxiv.org/abs/1511.08458>.
- [18] *Rectified Linear Unit (ReLU)*. URL: <https://www.giskard.ai/glossary/rectified-linear-unit-relu>.
- [19] SunEdition. *Graphs Dataset*. 2021. URL: <https://www.kaggle.com/datasets/sunedition/graphs-dataset>.
- [20] The pandas development team. *pandas*. Feb. 2020. DOI: 10.5281/zenodo.3509134. URL: <https://doi.org/10.5281/zenodo.3509134>.
- [21] The 365 Team. *What Is Cross-Entropy Loss Function?* 2023. URL: <https://365datascience.com/tutorials/machine-learning-tutorials/cross-entropy-loss/>.